Log and Exponential Functions

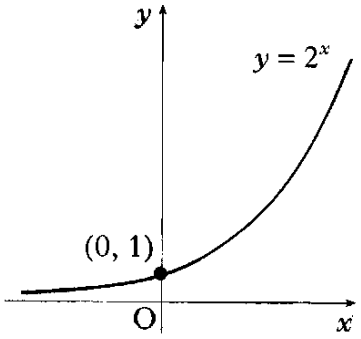
Exponential Graphs

2. (JAN) 02 P1

The diagram shows part of the graph of $y = 2^x$.

(*a*) Sketch the graph of $y = 2^{-x} - 8$. **2**

(*b*) Find the coordinates of the points where it crosses the x and y axes. **2**



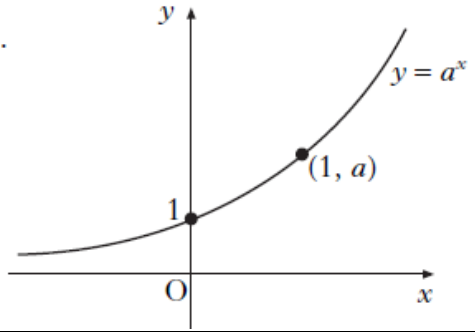
2007 P2

9. The diagram shows the graph of $y = a^x$, $a > 1$.

On separate diagrams, sketch the graphs of:

(*a*) $y = a^{-x}$; **2**

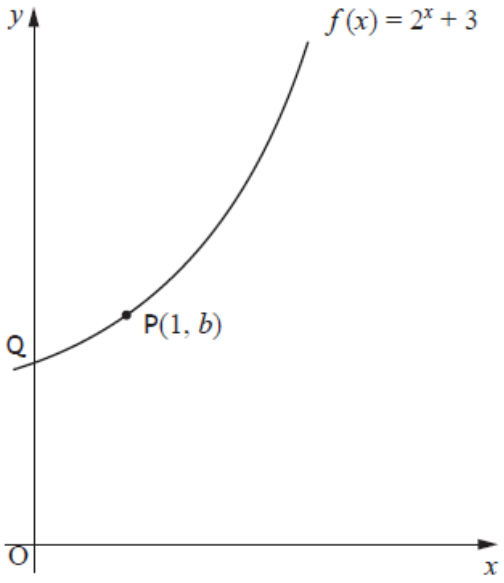
(*b*) $y = a^{1-x}$. **2**



2015 P1

13. The function $f(x) = 2^x + 3$ is defined on $\mathbb{R}$, the set of real numbers.

The graph with equation $y = f(x)$ passes through the point P$(1, b)$ and cuts the y-axis at Q as shown in the diagram.



(a) What is the value of b? **1**

(b) (i) Copy the above diagram.

On the same diagram, sketch the graph with equation $y = f^{-1}(x)$. **1**

(ii) Write down the coordinates of the images of P and Q. **3**

(c) R $(3,11)$ also lies on the graph with equation $y = f(x)$.

Find the coordinates of the image of R on the graph with equation $y = 4 - f(x + 1)$. **2**

Pre 2000 - Exponential Graphs

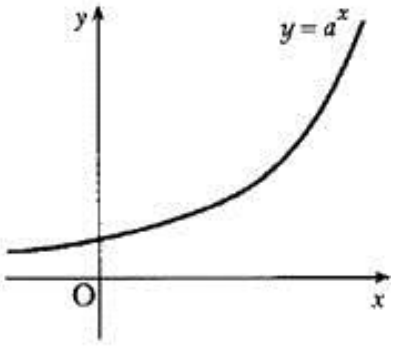
1

The diagram shows a sketch of part of the graph of $y = a^x, a > 1$.

(a) If $(1, t)$ and $(u, 1)$ lie on this curve, write down the values of t and u.

(b) Make a copy of this diagram and on it sketch the graph of $y = a^{2x}$.

(c) Find the coordinates of the point of intersection of $y = a^{2x}$ with the line $x = 1$.



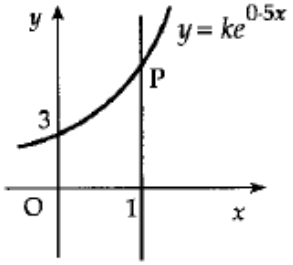
2, 2, 1

2

The diagram shows part of the graph of $y = ke^{0.5x}$.

(a) Find the value of k.

(b) The line with equation $x = 1$ intersects the graph at P. Find the coordinates of the point P.



1, 2

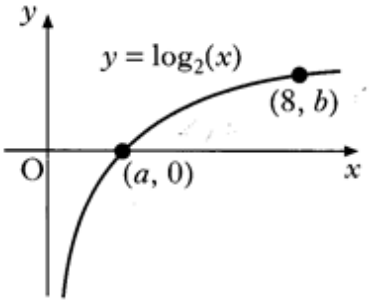
Log Graphs

2001 P1

10. The diagram shows a sketch of part of the graph of $y = \log_2(x)$.

(a) State the values of a and b. **1**

(b) Sketch the graph of $y = \log_2(x + 1) - 3$. **3**

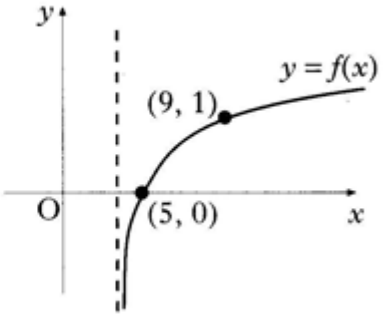


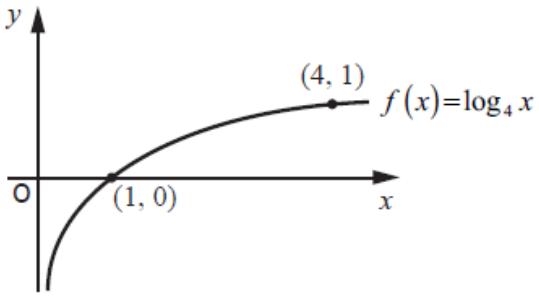
2005 P1

7. The function f is of the form $f(x) = \log_b(x - a)$. The graph of $y = f(x)$ is shown in the diagram.

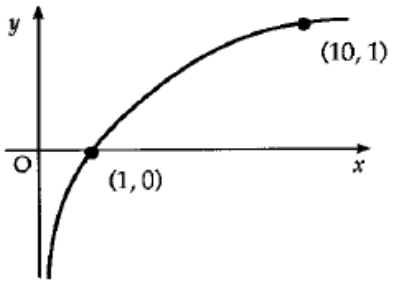
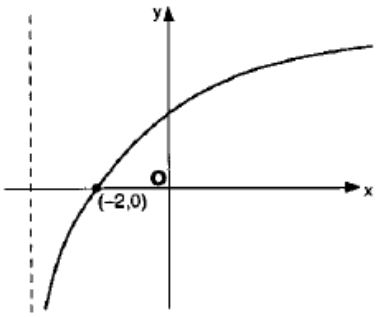
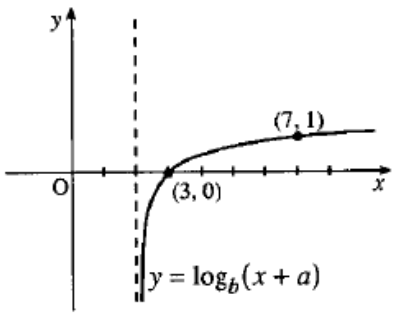
(a) Write down the values of a and b. **2**

(b) State the domain of f. **1**



2016 P1	**10.** The diagram below shows the graph of the function $f(x)=\log_4 x$, where $x>0$.  The inverse function, f^{-1}, exists. On the diagram in your answer booklet, sketch the graph of the inverse function. **2**

Pre 2000 - Log Graphs

1	Make a copy of this graph of $y=\log_{10} x$. On your copy, sketch the graph of $y=\log_{10}(x-2)$. **3** 
2	An incomplete sketch (not drawn to scale) of the graph of $\log_{10}(x+a)$ is shown. Find the value of a. **2** 
3	The diagram shows part of the graph of $y=\log_b(x+a)$. Determine the values of a and b. **3** 

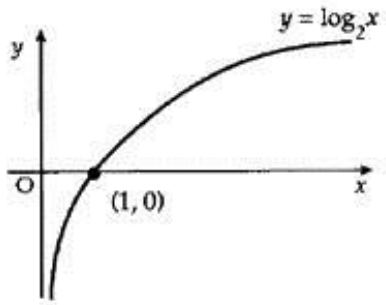
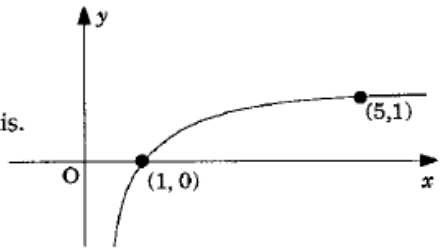
Laws of Logarithms

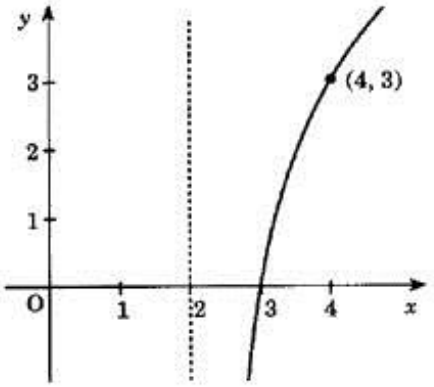
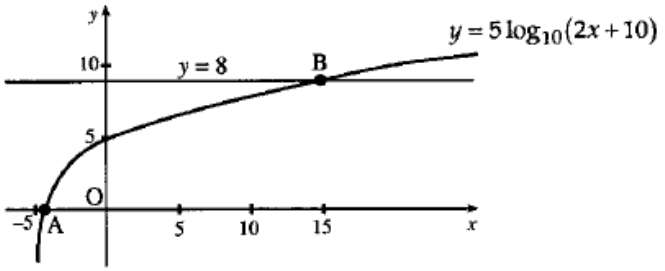
2000 P1	**B9.** Evaluate $\log_5 2 + \log_5 50 - \log_5 4$.	**3**
5. (JAN) 02 P1	If $\log_a p = \cos^2 x$ and $\log_a r = \sin^2 x$, show that $pr = a$.	**3**
2003 P1	**12.** Simplify $3\log_e(2e) - 2\log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers.	**4**
2015 P1	**6.** Evaluate $\log_6 12 + \frac{1}{3}\log_6 27$.	**3**

Log Graphs using Laws of Logs

2015 SP P2	**4.** (a) Express $y = \log_4 2x$ in the form $y = \log_4 x + k$, clearly stating the value of k.	**2**
	(b) Hence, or otherwise, describe the relationship between the graphs of $y = \log_4 2x$ and $y = \log_4 x$.	**1**
	(c) Determine the coordinates of the point where the graph of $y = \log_4 2x$ intersects the x-axis.	**2**
	(d) Sketch and annotate the graph of $y = f^{-1}(x)$, where $f(x) = \log_4 2x$.	**3**

Pre 2000 - Log Graphs using Laws of Logs

1	The diagram shows a sketch of the graph of $y = \log_2 x$. Make a rough copy of the diagram. On your copy, sketch the graph of $y = \log_2 2x$. 	3
2	The diagram shows a sketch of part of the graph of $y = \log_5 x$. (a) Make a copy of the graph of $y = \log_5 x$. On your copy, sketch the graph of $y = \log_5 x + 1$. Find the coordinates of the point where it crosses the x-axis. (b) Make a second copy of the graph of $y = \log_5 x$. On your copy, sketch the graph of $y = \log_5 \frac{1}{x}$. 	3 2

3	The diagram shows a sketch of the graph of $y = f(x)$ where $f(x) = a\log_2(x-b)$. Find the values of a and b.	 3
4	Part of the graph of $y = 5\log_{10}(2x+10)$ is shown in the diagram. This graph crosses the x-axis at the point A and the straight line $y = 8$ at the point B. Find algebraically the x-coordinates of A and B.	 4
5	(*a*) On the same diagram, sketch the graphs of $y = \log_{10} x$ and $y = 2 - x$ where $0 < x < 5$. Write down an approximation for the x-coordinate of the point of intersection. **(3)** (*b*) Find the value of this x-coordinate, correct to 2 decimal places. **(3)**	

Log Equations

2001 P1	**8.** Find x if $4\log_x 6 - 2\log_x 4 = 1$.	**3**
2002 P2	**7.** Find the x-coordinate of the point where the graph of the curve with equation $y = \log_3(x-2) + 1$ intersects the x-axis.	**3**
2004 P1	**9.** Solve the equation $\log_2(x+1) - 2\log_2(3) = 3$.	**4**
2005 P2	**7.** Solve the equation $\log_4(5-x) - \log_4(3-x) = 2$, $x < 3$.	**4**
2007 P2	**8.** The curve with equation $y = \log_3(x-1) - 2{\cdot}2$, where $x > 1$, cuts the x-axis at the point $(a, 0)$. Find the value of a.	**4**
2008 SP2 P1	**22.** Solve the equation $\log_x 8 + \log_x 4 = 5$.	**4**
2008 P1	**23.** Functions f, g and h are defined on suitable domains by $f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$. (*a*) Find expressions for $h(f(x))$ and $h(g(x))$. **3** (*b*) Hence solve $h(f(x)) - h(g(x)) = 3$. **5**	

2009 P2	**3.** (*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$. (ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4** (*b*) Solve $\log_2(x+3) + \log_2(x^2+5x-4) = 3$. **5**
2010 P2	**7.** (*a*) Given that $\log_4 x = P$, show that $\log_{16} x = \frac{1}{2}P$. **3** (*b*) Solve $\log_3 x + \log_9 x = 12$. **3**
2013 P2	**5.** Solve the equation $\log_5(3-2x) + \log_5(2+x) = 1$, where x is a real number. **4**
2015 OLD P1	**23.** Solve $\log_2(3x+7) = 3 + \log_2(x-1)$, $x > 1$. **4**
2016 P1	**14.** (a) Evaluate $\log_5 25$. **1** (b) Hence solve $\log_4 x + \log_4(x-6) = \log_5 25$, where $x > 6$. **5**
2017 P1	**12.** Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of a. **3**

Pre 2000 - Log Equations

1	Two sound intensities P_1 and P_2 are said to differ by n decibels when $n = 10\log_{10}\left(\frac{P_2}{P_1}\right)$ where P_1 and P_2 are measured in phons and $P_2 > P_1$. Rustling leaves have a typical sound intensity of 30 phons. If the sound intensity of a fire alarm siren is 6·5 decibels gretaer than rustling leaves, what is the sound intensity of the fire alrm system, measured in phons? **3**
2	Given $x = \log_5 3 + \log_5 4$, find algebraically the value of x. **4**
3	The point P (p, k) lies on the curve with equation $y = \ln x$. The point Q (q, k) lies on the curve with equation $y = \frac{1}{2}\ln x$. Find a relationship between p and q and hence find q when $p = 5$. **4**

Exponential Equations

2001 P2	**9.** Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later. If it takes one and half hours for the area of the forest fire to double, find the value of the constant k. **3**

6. (JAN) 02 P2	A researcher modelled the size N of a colony of bacteria t hours after the beginning of her observations by $N(t) = 950 \times (2{\cdot}6)^{0{\cdot}2t}$. (*a*) What was the size of the colony when observations began? **1** (*b*) How long does it take for the size of the colony to be multiplied by 10? **4**	
2003 P2	**11.** (*a*) (i) Sketch the graph of $y = a^x + 1$, $a > 2$. (ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$. **2** (*b*) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\frac{1}{a-1}\right)$. **3**	
2004 P2	**10.** The amount A_t micrograms of a certain radioactive substance remaining after t years decreases according to the formula $A_t = A_0e^{-0{\cdot}002t}$, where A_0 is the amount present initially. (*a*) If 600 micrograms are left after 1000 years, how many micrograms were present initially? **3** (*b*) The half-life of a substance is the time taken for the amount to decrease to half of its initial amount. What is the half-life of this substance? **4**	
2005 P2	**9.** The value V (in £ million) of a cruise ship t years after launch is given by the formula $V = 252e^{-0{\cdot}06335t}$. (*a*) What was its value when launched? **1** (*b*) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold? **4**	
2006 P2	**11.** It is claimed that a wheel is made from wood which is over 1000 years old. To test this claim, carbon dating is used. The formula $A(t) = A_0e^{-0{\cdot}000124t}$ is used to determine the age of the wood, where A_0 is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and t is the age of the wood in years. For the wheel it was found that $A(t)$ was 88% of the amount of carbon in a living tree. Is the claim true? **5**	
2008 SP1 P2	**6.** Radium decays exponentially and its half-life is 1600 years. If A_0 represents the amount of radium in a sample to start with and $A(t)$ represents the amount remaining after t years, then $A(t) = A_0e^{-kt}$. (*a*) Determine the value of k, correct to 4 significant figures. **3** (*b*) Hence find what percentage, to the nearest whole number, of the original amount of radium will be remaining after 3200 years. **2**	
2008 SP2 P2	**8.** The amount A_t micrograms of a certain radioactive substance remaining after t years decreases according to the formula $A_t = A_0e^{-0{\cdot}002t}$, where A_0 is the amount present initially. (*a*) If 600 micrograms are left after 1000 years, how many micrograms were present initially? **3** (*b*) The half-life of a substance is the time taken for the amount to decrease to half of its initial amount. What is the half-life of this substance? **4**	

2009 P2

6. The size of the human population, N, can be modelled using the equation $N = N_0 e^{rt}$ where N_0 is the population in 2006, t is the time in years since 2006, and r is the annual rate of increase in the population.

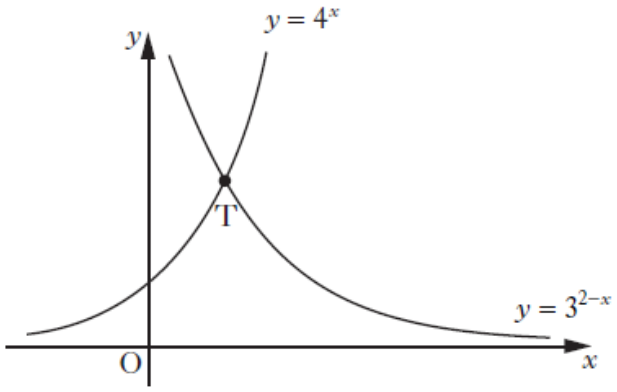
(*a*) In 2006 the population of the United Kingdom was approximately 61 million, with an annual rate of increase of 1·6%. Assuming this growth rate remains constant, what would be the population in 2020? **2**

(*b*) In 2006 the population of Scotland was approximately 5·1 million, with an annual rate of increase of 0·43%.

Assuming this growth rate remains constant, how long would it take for Scotland's population to double in size? **3**

2012 P2

7. The diagram shows the curves with equations $y = 4^x$ and $y = 3^{2-x}$.



The graphs intersect at the point T.

(*a*) Show that the x – coordinate of T can be written in the form $\frac{\log_a p}{\log_a q}$, for all $a > 1$. **6**

(*b*) Calculate the y – coordinate of T. **2**

2013 P2

9. The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

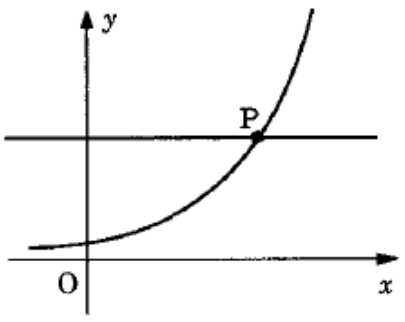
(*a*) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures. **4**

(*b*) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*? **3**

2015 SP P2	7. Given that $P(t) = 30e^{t-2}$ decide whether each of the statements below is true or false. Justify your answers. Statement A $P(0) = 30$. Statement B When $P(t) = 15$, the only possible value of t is 1·3 to one decimal place. 6
2015 EP P2	7. The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation. $$P_t = P_0 e^{-kt}$$ where: • P_0 is the initial concentration; • P_t is the concentration at time t; • t is the time, in days, after the application of the pesticide. Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value. (a) If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures. 4 On all *Xpesto* packaging, the manufacturer states that 80 days after application the concentration of *Xpesto* in the soil will have decreased by over 90%. (b) Is this statement correct? Justify your answer. 4
2016 P2	6. Scientists are studying the growth of a strain of bacteria. The number of bacteria present is given by the formula $$B(t) = 200\,e^{0.107t},$$ where t represents the number of hours since the study began. (a) State the number of bacteria present at the start of the study. 1 (b) Calculate the time taken for the number of bacteria to double. 4

Pre 2000 - Exponential Equations

1	The diagram shows part of the graph with equation $y = 3^x$ and the straight line with equation $y = 42$. These graphs intersect at P. Solve algebraically the equation $3^x = 42$, and hence write down, correct to 3 decimal places, the coordinates of P.  **4**
2	Medical researchers studying the growth of a strain of bacteria observe that the number of bacteria, present after t hours, is given by the formula $N(t) = 40e^{1.5t}$. (*a*) State the number of bacteria present at the start of the experiment. (*b*) How many minutes will the bacteria take to double in number? **1, 4**

3

A mug of tea cools according to the law $T_t = T_0 e^{-kt}$ where T_0 is the initial temperature and T_C is the temperature after t minutes. All temperatures are in °C.

(*a*) A particular mug of tea cooled from boiling point (100°C) to 75°C in a quarter of an hour. Calculate the value of k.

(*b*) By how many degrees will the temperature of this tea fall in the next quarter of an hour?

2, 3

4

The amount A grams of a radioactive substance at time t minutes is given by $A = A_0 e^{-kt}$ where A_0 is the initial amount of the substance and k is a constant.
In 3 minutes, 10 grams of the substance Bismuth are reduced to 9 grams through radioactive decay.

(*a*) Find the value of k.

The half-life of a substance is the length of time in which half the substance decays.

(*b*) Find the half-life of Bismuth.

3, 2

5

(*a*) For a particular radioactive substance the mass m (in grams) at time t (in years) is given by

$$m = m_0 e^{-0.02t}$$

where m_0 is the original mass.

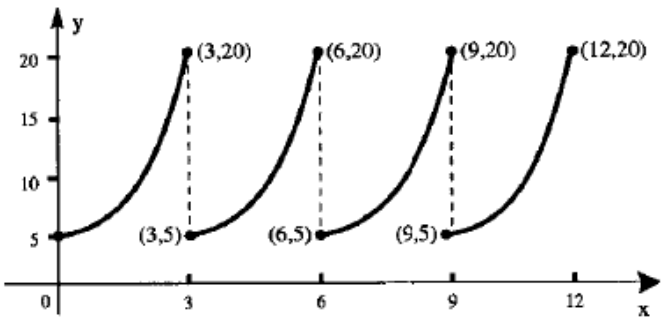
If the original mass is 500 grams, find the mass after 10 years. **(2)**

(*b*) The half-life of any material is the time taken for half of the mass to decay.
Find the half-life of this substance. **(3)**

(*c*) Illustrate **ALL** of the above information on a graph. **(3)**

6

A medical technician obtains this print-out of a wave form generated by an oscilloscope. The technician knows that the equation of the first branch of the graph (for $0 \le x \le 3$) should be of the form $y = ae^{kx}$.



(*a*) Find the values of a and k.

(*b*) Find the equation of the second branch of the curve (i.e. for $3 \le x \le 6$).

4, 1

7

(*a*) A tractor tyre is inflated to a pressure of 50 units.
Twenty-four hours later the pressure has dropped to 10 units.

If the pressure, P_t units, after t hours is given by the formula $P_t = P_0 e^{-kt}$, find the value of k, to three decimal places. **(5)**

(*b*) The tyre manufacturer advises that serious damage to the tyre will result if it is used when the pressure drops below 30 units.

If the farmer inflates the tyre to 50 units and drives the tractor for four hours, can the tractor be driven further without inflating the tyre and without risking serious damage to the tyre? **(4)**

8

The radioactive element carbon-14 is sometimes used to estimate the age of organic remains such as bones, charcoal, and seeds.
Carbon-14 decays according to a law of the form $y = y_0 e^{kt}$ where y is the amount of radioactive nuclei present at time t years and y_0 is the initial amount of radioactive nuclei.

(*a*) The half-life of carbon-14, i.e. the time taken for half the radioactive nuclei to decay, is 5700 years. Find the value of the constant k, correct to 3 significant figures. **(3)**

(*b*) What percentage of the carbon-14 in a sample of charcoal will remain after 1000 years? **(3)**

9

The intensity I_t of light is reduced as it passes through a filter according to the law $I_t = I_0 e^{-kt}$ where I_0 is the initial intensity and I_t is the intensity after passing through a filter of thickness t cm. k is a constant.

(*a*) A filter of thickness 4 cm reduces the intensity from 120 candle-power to 90 candle-power. Find the value of k.

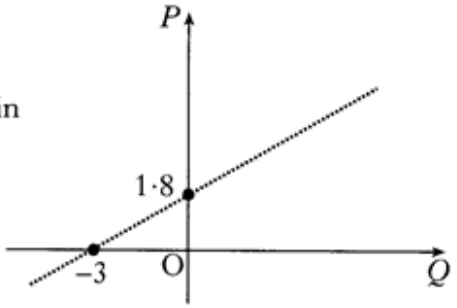
(*b*) The light is passed through a filter of thickness 10 cm. Find the percentage reduction in its intensity.

4, 3

Graphs with Log Axes

2000 P2

B11. The results of an experiment give rise to the graph shown.



(*a*) Write down the equation of the line in terms of P and Q. **2**

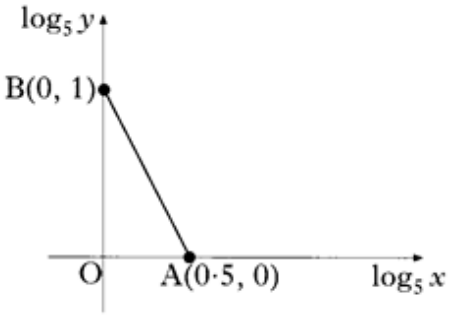
It is given that $P = \log_e p$ and $Q = \log_e q$.

(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. **4**

2002 P1

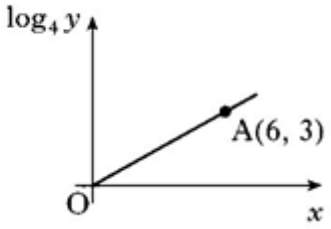
11. The graph illustrates the law $y = kx^n$.

If the straight line passes through A(0·5, 0) and B(0, 1), find the values of k and n. **4**



2006 P1

10. Two variables, x and y, are connected by the law $y = a^x$. The graph of $\log_4 y$ against x is a straight line passing through the origin and the point A(6, 3). Find the value of a. **4**



2007 P2

11. Two variables x and y satisfy the equation $y = 3 \times 4^x$.

(*a*) Find the value of a if $(a, 6)$ lies on the graph with equation $y = 3 \times 4^x$. **1**

(*b*) If $(-\frac{1}{2}, b)$ also lies on the graph, find b. **1**

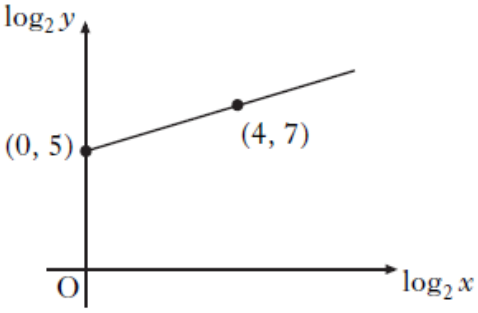
(*c*) A graph is drawn of $\log_{10} y$ against x. Show that its equation will be of the form $\log_{10} y = Px + Q$ and state the gradient of this line. **4**

2011 P2

5. Variables x and y are related by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points (0, 5) and (4, 7), as shown in the diagram.

Find the values of k and n. **5**

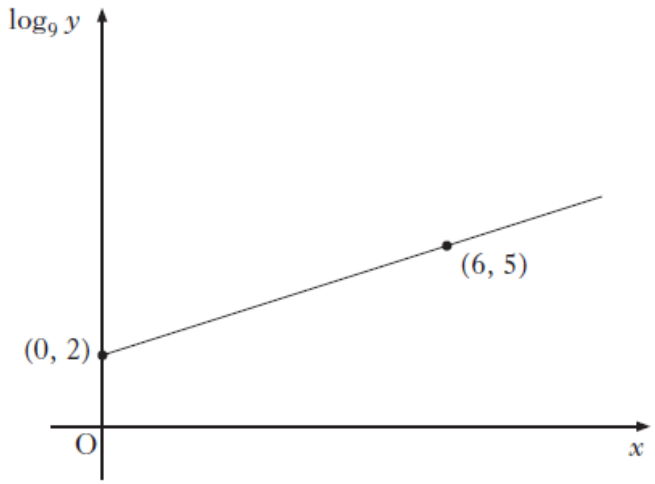


2014 P1

24. Two variables, x and y, are related by the equation

$$y = ka^x.$$

When $\log_9 y$ is plotted against x, a straight line passing through the points (0, 2) and (6, 5) is obtained, as shown in the diagram.

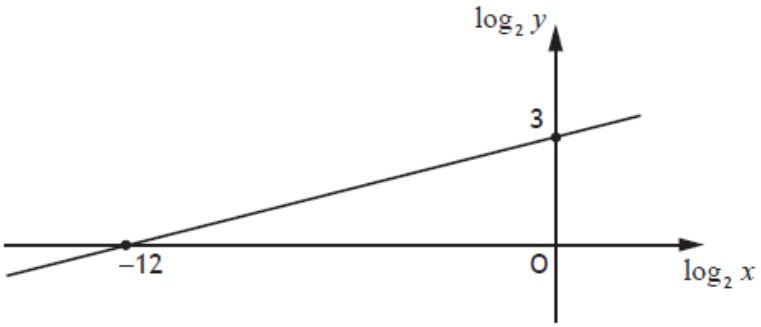


Find the values of k and a. 5

2017 P2

9. Two variables, x and y, are connected by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.



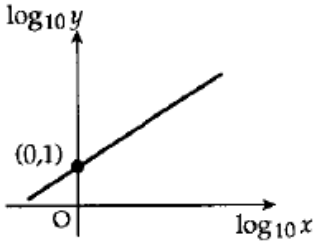
Find the values of k and n. 5

Pre 2000 - Graphs with Log Axes

1

As shown in the diagram, a set of experimental results gives a straight line graph when $\log_{10} y$ is plotted against $\log_{10} y$.
The straight line passes through (0, 1) and has a gradient of 2.

Express y in terms of x.



6

2

(*a*) The variables x and y are connected by a relationship of the form $y = ae^{bx}$ where a and b are constants. Show that there is a linear relationship between $\log_e y$ and x. **(3)**

(*b*) From an experiment some data was obtained. The table shows the data which lies on the line of best fit.

x	3.1	3.5	4.1	5.2
y	21 876	72 631	439 392	11 913 076

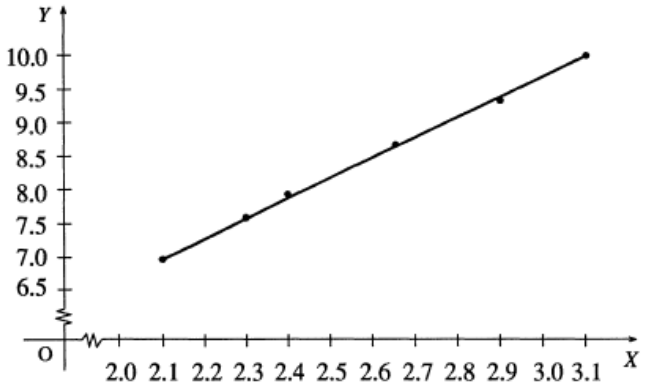
The variables x and y in the above table are connected by a relationship of the form $y = ae^{bx}$. Determine the values of a and b. **(6)**

3

Six spherical sponges were dipped in water and weighed to see how much water each could absorb. The diameter (x millimetres) and the gain in weight (y grams) were measured and recorded for each sponge. It is thought that x and y are connected by a relationship of the form $y = ax^b$.

By taking logarithms of the values of x and y, the table below was constructed.

X ($=\log_e x$)	Y ($=\log_e y$)
2.10	7.00
2.31	7.60
2.40	7.92
2.65	8.70
2.90	9.38
3.10	10.00



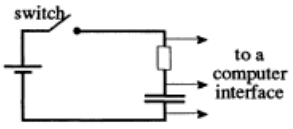
A graph was drawn and is shown above.

(*a*) Find the equation of the line in the form $Y = mX + c$. **(3)**

(*b*) Hence find the values of the constants a and b in the relationship $y = ax^b$. **(4)**

4

When the switch in this circuit was closed, the computer printed out a graph of the current flowing (I microamps) against the time (t seconds).This graph is shown in fig. 1.



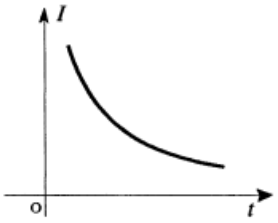


figure 1

In order to determine the equation of the graph shown in figure 1, values of $\log_e I$ were plotted against $\log_e t$ and the best fitting straight line was drawn as shown in figure 2.

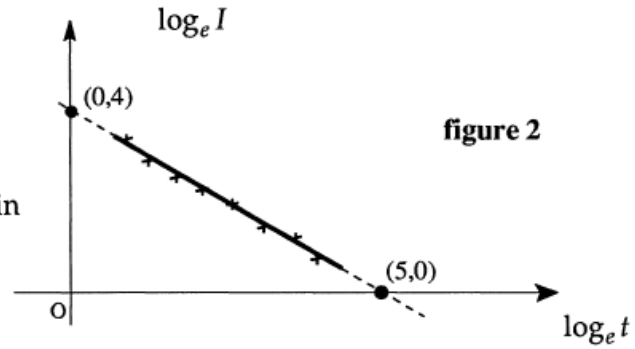


figure 2

(*a*) Find the equation of the line shown in figure 2 in terms of $\log_e I$ and $\log_e t$. **(3)**

(*b*) Hence or otherwise show that I and t satisfy a relationship of the form $I = kt^r$ stating the values of k and r. **(4)**